Pursuing the wrong options? Adjustment costs and the relationship between uncertainty and capital accumulation

Stephen R. Bond
Nuffield College and Department of Economics,
University of Oxford and Institute for Fiscal Studies

Måns Söderbom
Centre for the Study of African Economies,
Department of Economics, University of Oxford,
and Department of Economics, University of Gothenburg

Guiying Wu
Nuffield College and Department of Economics,
University of Oxford and the IFC, World Bank

September 2007

Abstract

This note shows that a higher level of uncertainty tends to reduce expected capital stock levels in a model with strictly convex adjustment costs. Simulations suggest that this negative impact of uncertainty on capital accumulation may be substantial. We also provide some intuition for this result.

JEL Classification: E22, D92, D81, C15.

Key words: Uncertainty, investment, capital accumulation.

Acknowledgement: We thank the ESRC for financial support under project RES-156-25-0006. Bond thanks the ESRC Centre for Public Policy at IFS for additional support. Söderbom thanks The Leverhulme Trust for additional support. Wu thanks the Enterprise Analysis Unit of the IFC, World Bank for additional support.

1 Introduction

The recent literature on uncertainty and investment has focused on the effects of 'real options' associated with non-convex forms of adjustment costs, such as (partial) irreversibility or fixed costs. The option to delay investment or disinvestment decisions, rather than to implement them in the current period, is valuable, and therefore influences current investment decisions, in models with non-convex adjustment costs and an elasticity of operating profits with respect to capital that is strictly less than unity. These options are more valuable in environments where firms are subject to greater uncertainty about future demand or profitability. This generates a relationship between uncertainty and investment behaviour, even for risk-neutral firms, that has been extensively analysed.[1] At a higher level of uncertainty, firms are less likely to invest in response to a given realisation of good news about their demand or profitability, as the option to 'wait and see' is more valuable.

Nevertheless the relationship between uncertainty and capital stock levels is much less clear in this class of model. From a development policy perspective, the impact of uncertainty on long run capital accumulation is likely to be more significant than the effects of uncertainty on short run investment dynamics.

Abel and Eberly (1999) characterise the relationship between uncertainty and expected capital stock levels analytically in a particular model with demand uncertainty and complete irreversibility, and no other sources of uncertainty or forms of adjustment costs. The more cautious response of investment to good news about demand is reflected in a 'user cost' effect, such that a higher threshold value of the marginal revenue product of capital is required to induce positive investment for firms subject to a higher level of uncertainty. All else equal this would result in lower capital stocks for firms in more uncertain environments. But all else is not equal. Working in the opposite direction is a 'hangover' effect, describing the

[1] See, for example, Dixit and Pindyck (1994) and Abel and Eberly (1996).

fact that firms subject to irreversibility may be stuck with more capital than they would like to have following the realisation of bad demand shocks. Firms facing a higher level of uncertainty will tend to experience larger negative demand shocks, leaving them with more excess capital in such periods. The sign of the relationship between uncertainty and average or expected capital stock levels depends on the net effect of these two opposing mechanisms, and is theoretically ambiguous.[2] Calculations reported by Abel and Eberly (1999) for their model also suggest that this net effect may be small. The expected level of the capital stock varies by only about 1 per cent over the range of values for the uncertainty parameter considered in their Figures 1-3.

In contrast, this note shows that in a model with strictly convex adjustment costs, a higher level of uncertainty tends to reduce expected capital stock levels, and this effect may be substantial. We simulate optimal investment decisions and track the evolution of optimal capital stocks in a discrete time model with a similar structure to that analysed by Abel and Eberly (1999), except that we consider more general forms of adjustment costs. This allows us to study how average capital stock levels vary with the level of uncertainty in various special cases of the model.

By construction, in the absence of adjustment frictions, the expected level of the capital stock is invariant to the level of demand uncertainty in the class of models we study here, so that any effects of uncertainty on average capital stock levels are attributable to different forms of adjustment costs. In the special case with complete irreversibility only, our numerical findings replicate the analytical results of Abel and Eberly (1999). However we find a strong negative relationship between uncertainty and average capital stock levels in the special case of the model with a standard form of quadratic adjustment costs. Intermediate results

[2] Caballero (1999) makes the same point more broadly. While firms facing higher uncertainty are more reluctant to invest in response to good news, they are also more reluctant to dis-invest in response to bad news. The net effect of uncertainty on expected capital stock levels is unclear.

are found for the cases of partial irreversibility and fixed adjustment costs.

The intuition for this result is as follows. Firms operating under uncertainty anticipate that future fluctuations in demand will require them to adjust their capital stocks. Given that capital stock adjustment is costly, this introduces a cost associated with using capital. The expected level of this cost can be reduced by substituting away from capital towards more flexible inputs. With strictly convex (i.e. increasing marginal) adjustment costs, this incentive to substitute away from capital is greater in environments with higher uncertainty, resulting in lower expected capital stock levels.

Section 2 describes the investment model that we study in this note. Section 3 presents the main results, and section 4 concludes.

2 Investment model

Following Abel and Eberly (1999), we assume that firms face isoelastic, downward-sloping, stochastic demand schedules of the form

$$Q_t = X_t P_t^{-\eta} \tag{1}$$

where Q_t is output, P_t is price and $-\eta < -1$ is the price elasticity of demand. The demand shift parameter X_t is stochastic and is the only source of uncertainty in the model. The log of this demand shift parameter follows a random walk with drift

$$\begin{aligned} x_t &= \ln X_t = x_{t-1} + \widetilde{\mu} + \varepsilon_t \qquad (2) \\ \varepsilon_t &\sim \text{iid } N(0, \sigma^2) \\ x_0 &= 0 \end{aligned}$$

which is the discrete time analogue of the geometric Brownian motion process considered in Abel and Eberly (1999). We follow Abel and Eberly (1999) in specifying $\widetilde{\mu} = \mu - 0.5\sigma^2$, so that the expected level of demand $E[X_t] = \mu t$ does not depend on the variance of the demand shocks (σ^2). That is, we consider the effects of mean-preserving spreads in the distribution of demand.

Firms produce output using capital and labour. Labour (L_t) is hired each period at the wage rate w, and is not subject to any adjustment costs. The firm inherits K_t units of capital from the past and purchases a further I_t units in period t. The purchase price of capital goods is normalised to unity. For numerical convenience, we assume that investment becomes productive in the current period, so the productive capital stock in period t is $(K_t + I_t)$. We follow Abel and Eberly (1999) in assuming that capital does not depreciate, so the capital stock evolves according to $K_{t+1} = K_t + I_t$. Investment also incurs adjustment costs $G(I_t, K_t)$, which are discussed further below.

As in Abel and Eberly (1999), we assume a constant returns to scale, non-stochastic Cobb-Douglas production function

$$Q_t = (K_t + I_t)^{\beta} L_t^{1-\beta}$$

Net revenue in period t is then given by

$$P_t Q_t - G(I_t, K_t) - I_t - wL_t$$

The firm's objective is to maximise the net present value of current and expected future net revenues. Following Abel and Eberly (1999), the optimal choice of the flexible labour input allows the net revenue function to be simplified to

$$hX_t^{\gamma}(K_t + I_t)^{1-\gamma} - G(I_t, K_t) - I_t$$

where

$$0 < \frac{1}{\eta} < \gamma = \frac{1}{1 + \beta(\eta - 1)} < 1$$

and

$$h = \left(\frac{1}{\gamma\eta}\right)^{\gamma\eta} (\gamma\eta - 1)^{\gamma\eta - 1} w^{1-\gamma\eta} > 0$$

We choose units of labour such that $h = 1$, giving the net revenue function

$$X_t^{\gamma}(K_t + I_t)^{1-\gamma} - G(I_t, K_t) - I_t \tag{3}$$

where $X_t^{\gamma}(K_t + I_t)^{1-\gamma} = P_t Q_t - wL_t$ denotes operating profits.

2.1 Adjustment costs

We depart from Abel and Eberly (1999) by allowing for more general forms of adjustment costs.

Partial irreversibility allows the price at which firms can sell units of capital (p^S) to be less than the price at which firms must buy units of capital, perhaps reflecting asymmetric information in the market for second hand capital goods (Akerlof, 1970). Since we have normalised the purchase price to unity, this can be represented by adjustment costs of the form

$$G(I_t) = -b_i I_t 1_{[I_t<0]}$$

where $1_{[I_t<0]}$ is an indicator equal to one if investment is strictly negative (i.e. the firm sells $-I_t$ units of capital) and equal to zero otherwise, and $b_i = 1 - p^S \geqslant 0$. For example, if $p^S = 0.8$ we have $b_i = 0.2$, indicating that the sale price is 20% lower than the purchase price. In general optimal investment may be positive or negative. Letting p^S approach zero, or letting b_i approach one, ensures that the firm never chooses to sell units of capital, and mimics investment behaviour under a complete irreversibilty constraint.

Fixed adjustment costs are paid if any investment or dis-investment is undertaken, and avoided if investment is zero. Letting the level of these fixed adjustment costs vary with the size of the firm, in proportion to operating profits, these can be represented by adjustment costs of the form

$$G(I_t, K_t) = b_f 1_{[I_t \neq 0]} X_t^{\gamma}(K_t + I_t)^{1-\gamma}$$

where $1_{[I_t \neq 0]}$ is an indicator equal to one if investment is non-zero.

Strictly convex adjustment costs are increasing at the margin as the firm undertakes additional investment (or dis-investment). We consider a standard quadratic adjustment cost function which is homogeneous of degree one in I_t and K_t, again allowing the level of these quadratic adjustment costs to vary with the size of the firm

$$G(I_t, K_t) = \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t \tag{4}$$

Our model allows for these three forms of adjustment costs, specifying the adjustment cost function to be

$$G(I_t, K_t) = -b_i I_t 1_{[I_t<0]} + b_f 1_{[I_t\neq 0]} X_t^{\gamma}(K_t + I_t)^{1-\gamma} + \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t \tag{5}$$

2.2 Dynamic optimisation

The firm is assumed to have a discount rate of r per period, or a discount factor of $\phi = \frac{1}{1+r}$. Investment in period t is chosen to maximise the present discounted value of current and expected future net revenues, where expectations are taken over the distribution of future demand shocks. This investment decision can be represented as the solution to a dynamic optimisation problem defined by the stochastic Bellman equation

$$V_t(X_t, K_t) = \max_{I_t} \Pi(X_t, K_t; I_t) + \phi E_t\left[V_{t+1}(X_{t+1}, K_{t+1})\right]$$

where V_t is the value of the firm in period t, $E_t[V_{t+1}]$ is the expected value of the firm in period $t+1$ conditional on information available in period t, and

$$\Pi(X_t, K_t; I_t) = X_t^{\gamma}(K_t + I_t)^{1-\gamma} - G(I_t, K_t) - I_t$$

is net revenue in period t, as in equation (3). The two state variables are the capital stock K_t and the level of demand X_t, with equations of motion defined above.

Given our specification for adjustment costs, there is no analytical solution that describes the optimal level of investment I_t as a function of the state variables X_t and K_t. However we can use numerical stochastic dynamic programming methods to simulate these optimal investment decisions. The model outlined here is closely related to that considered in Bloom (2006) and Bloom et al. (2007), and we use a similar algorithm to generate the simulated investment data. Further details are given in Appendix A.

The investment decision rule for this problem allows investment rates (I_t/K_t) to be considered as a function of (X_t/K_t). The ratio of the two state variables reflects the imbalance between the productive capital that the firm would like to have, given the realisaton of the level of demand in period t, and the capital stock that the firm has inherited from the previous period. These decision rules have the expected properties in four special cases of the model, corresponding to no adjustment costs, partial irreversibility only, fixed costs only, and quadratic costs only.

In the absence of adjustment costs, the optimal level of productive capital $(K_t + I_t)$ is proportional to the level of demand (X_t), and investment rates are a linear function of the imbalance (X_t/K_t).

With partial irreversibility only, there is a 'region of inaction', or a range of values of (X_t/K_t) for which optimal investment is zero. Outside this range, the firm chooses the minimum level of investment or dis-investment required to keep the marginal revenue product of capital below an upper bound or above a lower bound (barrier control).

With fixed costs only, there is also a region of inaction. Very low levels of investment or dis-investment are not optimal in the presence of fixed costs, so outside this region of inaction the firm chooses rates of investment or dis-investment that return the marginal revenue product of capital to an interior point between upper and lower thresholds (jump control).

With quadratic adjustment costs only, there is no region of inaction. The optimal investment rate varies monotonically with the imbalance (X_t/K_t). Since large adjustments are penalised, the optimal investment rate is less sensitive to this imbalance that it would be in the absence of adjustment costs. The optimal response to a permanent demand shock takes the form of a sequence of smaller adjustments.

3 The relationship between uncertainty and expected capital stocks

The investment model outlined in section 2 is fully parametric. Once we specify values for the parameters of the demand process given in equation (2) (i.e. μ and σ), the parameters of the adjustment cost function given in (5) (i.e. b_i, b_f and b_q), the elasticity of operating profits with respect to productive capital $(1 - \gamma)$ and the discount rate (r), we can use the numerical solution to the investment decision problem described above and outlined in Appendix A to generate simulated data on investment and capital stocks for hypothetical panels of firms. We simply draw different histories of the demand shocks (ε_t) from the distribution specified in (2), and track each firm's optimal investment decisions in response to these realisations of the stochastic demand process.

The special case of our model with complete irreversibility $(b_i = 1)$ and no other forms of adjustment costs $(b_f = b_q = 0)$ is a discrete time version of the model analysed by Abel and Eberly (1999). In this section we use the same parameter values that were used by Abel and Eberly (1999) to quantify the relationship between the level of uncertainty (σ) and the expected level of the capital stock under complete irreversibility relative to the expected level of the capital stock in the absence of adjustment costs (i.e. $E[K_t]/E[K_t^*]$, where K_t^* denotes the optimal capital stock in the frictionless case). These values are $\mu = 0.029, \gamma = 0.2519$ and $r = 0.05$.

We construct a simulated counterpart to Figure 1 in Abel and Eberly (1999) by generating simulated data on capital stocks for hypothetical panels of 1,000,000 firms at different values of the uncertainty parameter (σ). In each case we compute the optimal capital stock that would be chosen in the absence of adjustment costs (K_t^*) as well as the optimal capital stock that is chosen in the case of complete irreversibility (K_t), using the same realisations of the demand shocks. For each level of uncertainty, we calculate the mean level of K_t and K_t^* for the sample of 1,000,000 hypothetical firms in the same reference year. The reference year is chosen so that any effect of the initialisation of our simulations has become negligible, and we check that similar results are found for later reference years.[3] Similar results were also obtained using the analytical expression for $E[K_t^*]$ in place of the simulated means $\overline{K}_t^*$.[4]

Figure 1 plots the ratio $\overline{K}_t/\overline{K}_t^*$ against σ. The dashed line shows the actual estimates of $\overline{K}_t/\overline{K}_t^*$, which fluctuate somewhat as the result of numerical inaccuracies.[5] The solid line fits a simple polynomial regression through these points to illustrate the general pattern. This reproduces the main features of Figure 1 in Abel and Eberly (1999). At very low levels of uncertainty, the presence of complete irreversibility has almost no effect on the expected level of the capital stock. Indeed as $\sigma \to 0$, complete irreversibility becomes irrelevant for firms that are experiencing certain, positive growth in demand. As the level of demand uncertainty increases, the expected level of the capital stock under complete irreversibility initially increases relative to the expected level of the capital stock in the frictionless case. Over this range the 'hangover' effect described in Abel and Eberly (1999) dominates the 'user cost' effect, so that on average we find higher capital stock levels in the simulations with higher levels of uncertainty. This effect peaks at values of σ around 0.16-0.18, where the average capital stock level is about 1 per cent higher than it would be in the absence of either irreversibility or demand

[3] Figure 1 reports the results using $t = 100$. Details of the initialisation of our simulations are given in Appendix A. Figure 1 in Abel and Eberly (1999) considers the case where $t \to \infty$.

[4] It should be noted that $E[K_t^*]$ does not vary with σ in the model we consider here. This reflects the properties that K_t^* is proportional to X_t, and $E[X_t]$ is invariant to σ. The former property in turn reflects the linear homogeneity of operating profits in X_t and $(K_t + I_t)$ (see (3)), which follows from specifying uncertainty in the quantity of output demanded at any given price (see (1)). This has the advantage that any effects of uncertainty on $E[K_t]/E[K_t^*]$ are attributable to the effects of adjustment costs on $E[K_t]$. However it should be noted that this is restrictive. In particular there are no Jensen's inequality effects of the kind studied by Hartman (1972), Abel (1983) and Caballero (1991) present in the model we study here.

[5] These fluctuations are not reduced by increasing the number of firms in our generated samples. This suggests that they reflect inaccuracy in our numerical approximations to the optimal investment decisions, rather than the sample means simply providing inaccurate estimates of expected values.

uncertainty. For higher values of the uncertainty parameter, the expected capital stock under complete irreversibility is then decreasing in the level of uncertainty. For values of σ in the range 0.22-0.23, the 'user cost' effect dominates the 'hangover' effect, and we find average capital stock levels in the presence of complete irreversibility that are about 0.5 per cent lower than they would be in the absence of either irreversibility or demand uncertainty.[6]

This confirms the analytical results in Abel and Eberly (1999) and suggests that our numerical results are in the right ballpark. In the special case of the model with complete irreversibility and no other forms of adjustment costs, a higher level of uncertainty may result in either higher or lower average capital stock levels, depending on whether the 'hangover' effect or the 'user cost' effect dominates. At least using the parameter values considered in Abel and Eberly (1999), the net effect of variation in the level of uncertainty on expected capital stock levels also appears to be small. The average level of the capital stock varies by less than 2 per cent over the whole range of values considered for the uncertainty parameter.

Figure 2 considers a specification with partial irreversibility rather than complete irreversibility, and no other forms of adjustment costs. Here we have $b_i = 0.1$ and $b_f = b_q = 0$. All other parameter values are the same as those used to generate Figure 1. With partial irreversibility, the relationship between demand uncertainty and expected capital stock levels has a similar shape to that shown under complete irreversibility in Figure 1, but the magnitudes are different. At low levels of uncertainty, the expected capital stock level is again increasing in the standard deviation of the demand shocks. The peak again has average capital stock levels under partial irreversibility that are about 1 per cent higher than average capital stock levels under no adjustment costs, but this peak occurs at lower values of σ

[6] These are the highest levels of demand uncertainty that we can consider in this model. The marginal revenue product of capital has a non-degenerate ergodic distribution only under the restriction $\mu > 0.5\sigma^2$. See equation (7) in Abel and Eberly (1999).

around 0.1. At higher levels of uncertainty, the expected capital stock is again decreasing in the standard deviation of the demand shocks. For values of σ above 0.15, average capital stock levels under partial irreversibility are lower than those that would be chosen in the absence of adjustment costs. For values of σ above 0.2, the effect of uncertainty under partial irreversibility is to reduce average capital stock levels by about 5 per cent. The 'hangover' effect appears to be less important in the case of partial irreversibility, where firms can choose to adjust capital stocks downwards, than it is under complete irreversibility.

Figure 3 considers a specification with fixed adjustment costs only. Here we have $b_f = 0.05$ and $b_i = b_q = 0$. One important difference is that the presence of fixed adjustment costs affects optimal capital stock levels even in the case of complete certainty (i.e. as $\sigma \rightarrow 0$). Firms with deterministic positive demand growth will want to have growing capital stocks, which requires positive investment. Under fixed costs, this adjustment will take the form of infrequent, large investments, implying that some adjustment costs will be paid. Firms can reduce the expected level of these adjustment costs by using less capital and more labour, so we find that expected capital stock levels in the presence of fixed adjustment costs are lower than they would be in the frictionless case, even for very low values of the uncertainty parameter (σ). For example, at $\sigma = 0.05$ we find that average capital stock levels are about 3.5 per cent lower. As we found for the specifications with irreversibility, the expected capital stock level initially increases with the level of uncertainty. In this case, average capital stock levels peak at values of σ in the range 0.07-0.09. For higher levels of uncertainty, the expected capital stock is again decreasing with the level of uncertainty. For values of σ above 0.2, the effect of uncertainty in the presence of these fixed adjustment costs is also to reduce average capital stock levels by about 5 per cent. This is similar to the effect found in the specification with partial irreversibility, and considerably larger than the effect under complete irreversibility.

Figure 4 considers a specification with quadratic adjustment costs only. Here we have $b_q = 0.5$ and $b_i = b_f = 0$. Again the presence of quadratic adjustment costs reduces average capital stock levels even in an environment with perfect certainty, due to the positive trend growth in the level of demand. For $\sigma = 0.05$ we find that average capital stock levels are about 5 per cent lower with quadratic adjustment costs than they would be in the absence of adjustment costs. In this case, we find that average capital stock levels fall monotonically as we consider higher levels of demand uncertainty. The magnitude of this effect is also much greater than we found with partial irreversibility or with fixed adjustment costs. For $\sigma = 0.15$ we find that average capital stock levels are about 10 per cent lower in this specification, and for values of σ above 0.2 the expected level of the capital stock with this form of quadratic adjustment costs is around 30 per cent lower than in the frictionless case.

This illustrates the two main findings of this analysis. In a dynamic investment model with strictly convex adjustment costs only, we find that a higher level of uncertainty tends to reduce the expected level of the capital stock. Moreover this impact of uncertainty on capital accumulation can be quantitatively significant. The effect of uncertainty on average capital stock levels is an order of magnitude larger in our specification with quadratic adjustment costs than in our specification with complete irreversibility. Intermediate results are found for specifications with partial irreversibility or with fixed adjustment costs.

3.1 Some intuition

We can provide some intuition for this relationship between uncertainty and average capital stock levels in the presence of quadratic adjustment costs by analogy with the effect of such adjustment costs on optimal investment decisions in a steady state setting. Suppose for simplicity that firms rent units of capital at the rental cost of c per period. The net revenue function is then

$$X_t^{\gamma}(K_t + I_t)^{1-\gamma} - G(I_t, K_t) - c(K_t + I_t)$$

rather than the form given in equation (3). Consider now a steady state in which the capital stock K_t grows at the constant rate $\mu > 0$. Using the equation of motion for the capital stock $K_{t+1} = K_t + I_t$, this implies that $I_t = \mu K_t$ or $I_t/K_t = \mu$. Now if the firm is subject to quadratic adjustment costs of the form given in (4), this implies that $G(I_t, K_t) = (b_q/2)\mu^2 K_t$ or $(b_q/2)\lambda(K_t + I_t)$ where $\lambda = \mu^2/(1+\mu)$. The net revenue function can then be written as

$$X_t^\gamma (K_t + I_t)^{1-\gamma} - \left(c + \frac{b_q \lambda}{2}\right)(K_t + I_t).$$

Compared to the case of no adjustment costs ($b_q = 0$), it is clear that the firm facing this form of quadratic adjustment costs ($b_q > 0$) acts *as if* it faces a higher cost of capital in this steady state with positive growth ($\mu > 0 \Rightarrow \lambda > 0$). This implies that it will choose a lower level of the capital stock along its steady state growth path (i.e. compared to the case of no adjustment costs, there is a parallel downward shift in the optimal steady state path for the capital stock). This illustrates how forward-looking firms that anticipate future costs associated with adjusting their capital stocks may be induced to substitute away from capital towards the flexible labour input.

This result explains why firms subject to quadratic adjustment costs choose lower capital stocks in an environment with certain, positive demand growth, and accounts for the lower average capital stock levels found in the simulations with very low values of σ in Figure 4. The intuition that forward-looking firms may substitititute away from capital towards labour if they anticipate having to pay future costs to adjust their capital stocks suggests that uncertainty about the level of future demand will have a similar effect. Firms operating in uncertain environments anticipate that future fluctuations in demand will require them to adjust their capital stocks, which implies a cost associated with using capital. The expected level of this cost can be reduced by substituting away from capital towards more flexible inputs. With strictly convex (i.e. increasing marginal) adjustment costs, this incentive to substitute away from capital is greater in environments with a higher level of uncertainty. This accounts for the monotonic relationship between uncertainty and expected capital stock levels that we find in our simulation with quadratic adjustment costs. Our simulation suggests that this channel could generate a quantitatively significant negative impact of uncertainty on capital accumulation.

4 Conclusions

This note shows that a higher level of uncertainty tends to reduce expected capital stock levels in a model with strictly convex adjustment costs. Our model is a discrete-time version of that considered by Abel and Eberly (1999), except that we consider more general forms of adjustment costs. Our numerical simulations replicate the key features of their analytical results for the special case with complete irreversibility and no other forms of adjustment costs. Using instead a standard form of quadratic adjustment costs, we find that the negative impact of uncertainty on capital accumulation can be substantial.

In a companion paper we estimate structural parameters of a closely related model using data on firms in several developing countries. For most samples we find that quadratic adjustment costs play an important role in our estimated adjustment cost functions. As a result, counterfactual simulations suggest that reducing the level of uncertainty faced by firms in these countries could induce them to operate with substantially higher capital stocks. These findings are described in detail in Bond, Söderbom and Wu (2007).

References

[1] Abel, Andrew B., 1983, "Optimal investment under uncertainty", *American Economic Review*, vol. 73, pp. 228-233.

[2] Abel, Andrew B. and Janice C. Eberly, 1996, "Optimal investment with costly reversibility", *Review of Economic Studies*, vol. 63, pp. 581-593.

[3] Abel, Andrew B. and Janice C. Eberly, 1999, "The effects of irreversibility and uncertainty on capital accumulation", *Journal of Monetary Economics*, vol. 44, no. 3, pp. 339-377.

[4] Akerlof, George A., 1970, "The market for 'lemons': quality uncertainty and the market mechanism", *The Quarterly Journal of Economics*, vol. 84, no. 3, pp. 488-500.

[5] Bloom, Nick, 2000, "The dynamic effects of real options and irreversibility on investment and labour demand", Working Paper no. W00/15, Institute for Fiscal Studies, London.

[6] Bloom, Nick, 2006, "The impact of uncertainty shocks: firm level estimation and a 9/11 simulation", Mimeo, Department of Economics, Stanford University.

[7] Bloom, Nick, Stephen R. Bond and John Van Reenen, 2007, "Uncertainty and investment dynamics", *Review of Economic Studies*, vol. 74, pp. 391-415.

[8] Bond, Stephen R., Måns Söderbom and Guiying Wu, 2007, "Uncertainty and capital accumulation: empirical evidence for African and Asian firms", Mimeo, Department of Economics, University of Oxford.

[9] Caballero, Ricardo J., 1991, "On the sign of the investment-uncertainty relationship", *American Economic Review*, vol. 81, pp. 279-288.

[10] Caballero, Ricardo J., 1999, "Aggregate investment", in J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics vol. 1B*, North Holland.

[11] Dixit, Avinash K. and Robert S. Pindyck, 1994, *Investment Under Uncertainty*, Princeton University Press.

[12] Hartman, Richard, 1972, "The effect of price and cost uncertainty on investment", *Journal of Economic Theory*, vol. 5, pp. 258-266.

Appendix A: Algorithm

This appendix describes the numerical optimisation procedures used to solve the model and generate the simulated investment data.

The value of the firm is given by the Bellman equation

$$V_t(X_t, K_t) = \max_{I_t} \Pi(X_t, K_t; I_t) + \phi E_t \left[V_{t+1}(X_{t+1}, K_{t+1})\right] \tag{A1}$$

subject to the capital evolution constraint

$$K_{t+1} = I_t + K_t, \tag{A2}$$

and law of motion for demand

$$\frac{dX_t}{X_t} = \mu dt + \sigma dz \tag{A3}$$

Define $x_t = \ln X_t$. According to Ito's Lemma

$$dx_t = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dz \tag{A4}$$

The discretised version of (A4) is

$$x_t = x_{t-1} + \widetilde{\mu} + \sigma e_t$$

or equivalently

$$X_t = X_{t-1} \exp\left(\widetilde{\mu} + \sigma e_t\right) \tag{A5}$$

where $\widetilde{\mu} = \mu - \frac{1}{2}\sigma^2$, and $e_t \sim iidN\left(0, 1\right)$.

We solve this dynamic program numerically using value function iteration. Demand X_t and the beginning-of-period capital stock K_t are the state variables. Investment I_t, or equivalently the end-of-period capital stock $K_t + I_t$, is the control variable. In order to use value function iteration, state and control variables must be stationary. This is achieved by a normalisation of the problem suggested by Bloom (2006).

In the absence of adjustment costs, we can derive an analytical solution to (A1) that has the form

$$\frac{I_t}{K_t} = c_1 \left(\frac{X_t}{K_t}\right) - 1 \tag{A6}$$

which implies that the frictionless optimal capital stock (K_t^*) can be written as

$$K_t^* = c_2 X_t \tag{A7}$$

where $c_1 = \left[(1-\gamma)/(1-\phi)\right]^{1/\gamma}$, and $c_2 = c_1/\exp(\mu)$.

(A7) indicates that the ratio X_t/K_t^* is constant. Bloom (2000) shows that the ratio K_t^*/K_t is stationary or, equivalently, that $\ln K_t^*$ and $\ln K_t$ are cointegrated. Hence the ratio X_t/K_t is also stationary.

Noting that the revenue function and the adjustment cost function are both homogenous of degree one in (X_t, I_t, K_t), we can rewrite (A1) as

$$K_t V_t(g_t) = \max_{i_t} K_t \Pi(g_t; i_t) + \phi K_{t+1} E_t \left[V_{t+1}(g_{t+1})\right] \tag{A8}$$

where $g_t = X_t/K_t$ and $i_t = I_t/K_t$. Dividing by K_t on both sides of (A8) and using (A2), we get the normalised Bellman equation

$$V_t(g_t) = \max_{i_t} \Pi(g_t; i_t) + \phi\,(1+i_i)\, E_t \left[V_{t+1}(g_{t+1})\right] \tag{A9}$$

Now define $\widetilde{g}_t = X_t/K_{t+1}$. Then g_t, i_t and $\widetilde{g}_t$ are linked by the law of motion

$$i_t = \frac{g_t}{\widetilde{g}_t} - 1 \tag{A10}$$

Therefore (A9) can be rewritten as

$$V_t(g_t) = \max_{\widetilde{g}_t} \Pi(g_t; \widetilde{g}_t) + \phi\,(g_t/\widetilde{g}_t)\, E_t \left[V_{t+1}(g_{t+1})\right] \tag{A11}$$

Now in this formulation, g_t is the state variable and $\widetilde{g}_t$ is the control variable. Based on (A7), we define the support of g_t to be $g0 \in [\exp(-\ln c_2 - 5\sigma)$, $\exp(-\ln c_2 + 5\sigma)]$, and we discretise this state space using 200 grid points. Since conditional expectations need to be formed based on g_t, we extrapolate the state space $g0$ on both left and right sides by 50%. Conditional expectations are then calculated based on the extended transition matrix according to the normal CDF.

Since the (normalised) net revenue function is strictly concave in g_t for any $0 < \gamma < 1$, and the set of constraints $g0$ is compact and convex, there must exist a unique solution to the dynamic program (A11). To begin the value function iteration, we start with an arbitrary initial guess $V(g_t)_{[0]}$. For each g_t, we search along the state space $g0$ for the optimal policy rule $\widetilde{g}_t$, or equivalently i_t, which would maximise the value of the firm $V(g_t)_{[1]}$. We then use $V(g_t)_{[1]}$ to update $V(g_t)_{[0]}$ and repeat this procedure until the difference between $V(g_t)_{[j-1]}$ and $V(g_t)_{[j]}$ is within our tolerance $1e-8$. At this point, there is convergence and we have found the optimal solution $\widetilde{g}_t^* = f(g_t)$, or equivalently $i_t^* = f(g_t)$.

We use this numerical solution to the model to generate simulated panel data. We endow all simulated firms with the initial condition $X_{i0} = 1$ and $K_{i1} = c2$, i.e. $\widetilde{g}_{i0} = X_{i0}/K_{i1} = 1/c2$. According to (A5), $g_{i1} = X_{i1}/K_{i1} = \exp(\widetilde{\mu} + \sigma e_{i1})\,\widetilde{g}_{i0}$.

We then find the optimal investment rate i_{i1} using the policy rule derived above for each firm i in the first period 1. Then according to (A2), $K_{i2} = K_{i1}(1 + i_{i1})$, which updates g_{i1} into $\widetilde{g}_{i1}$. In all subsequent periods, $\widetilde{g}_{i,t-1}$ becomes g_{it} when X_{it} evolves exogenously according to (A5). For given g_{it}, optimal investment rates i_{it} are found from our numerical solution for each firm i in each period t. Then g_{it} becomes $\widetilde{g}_{i,t}$ when $K_{i,t+1}$ evolves endogenously according to (A2).

The actual level of investment is easily recovered as $I_{it} = i_{it}K_{it}$. With the simulated data for I_{it} and K_{it}, and the assumed paramter values, variables of interest such as adjustment costs and revenue can be easily calculated.

Figure 1: The Effect of Uncertainty on Average Capital Stock Levels: Complete Irreversibility Only

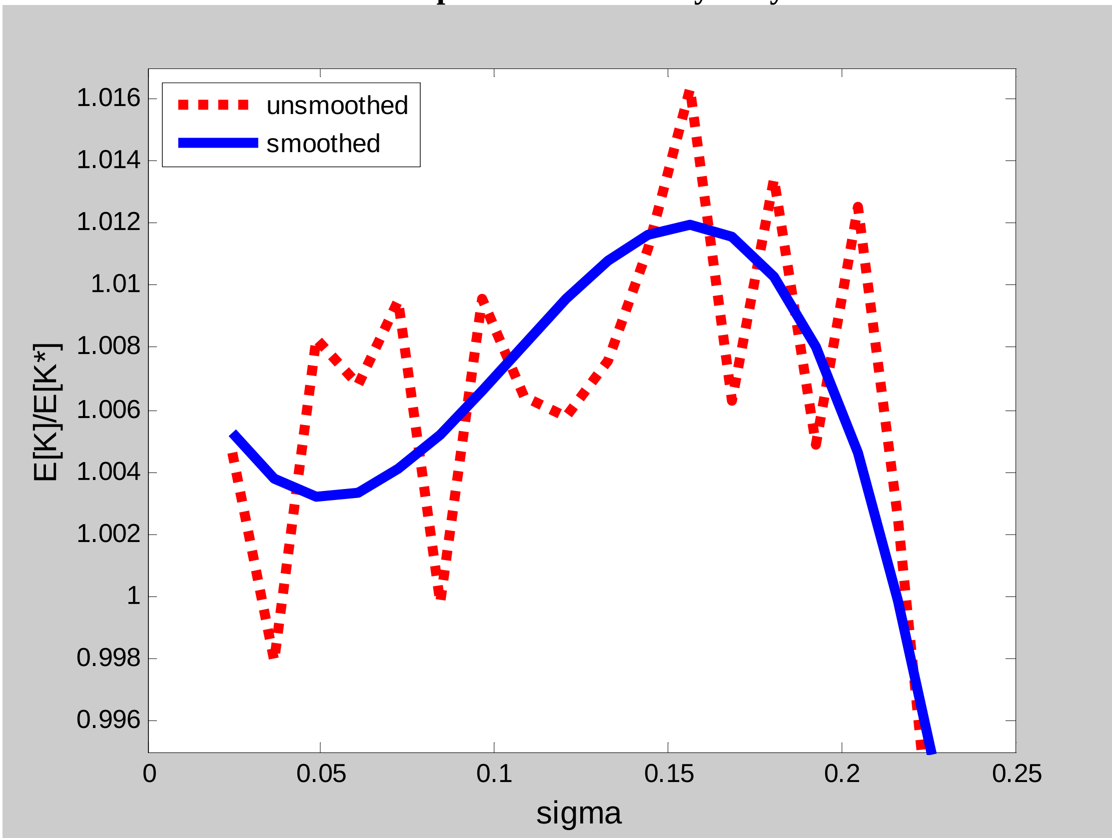

Figure 2: The Effect of Uncertainty on Average Capital Stock Levels: Partial Irreversibility Only

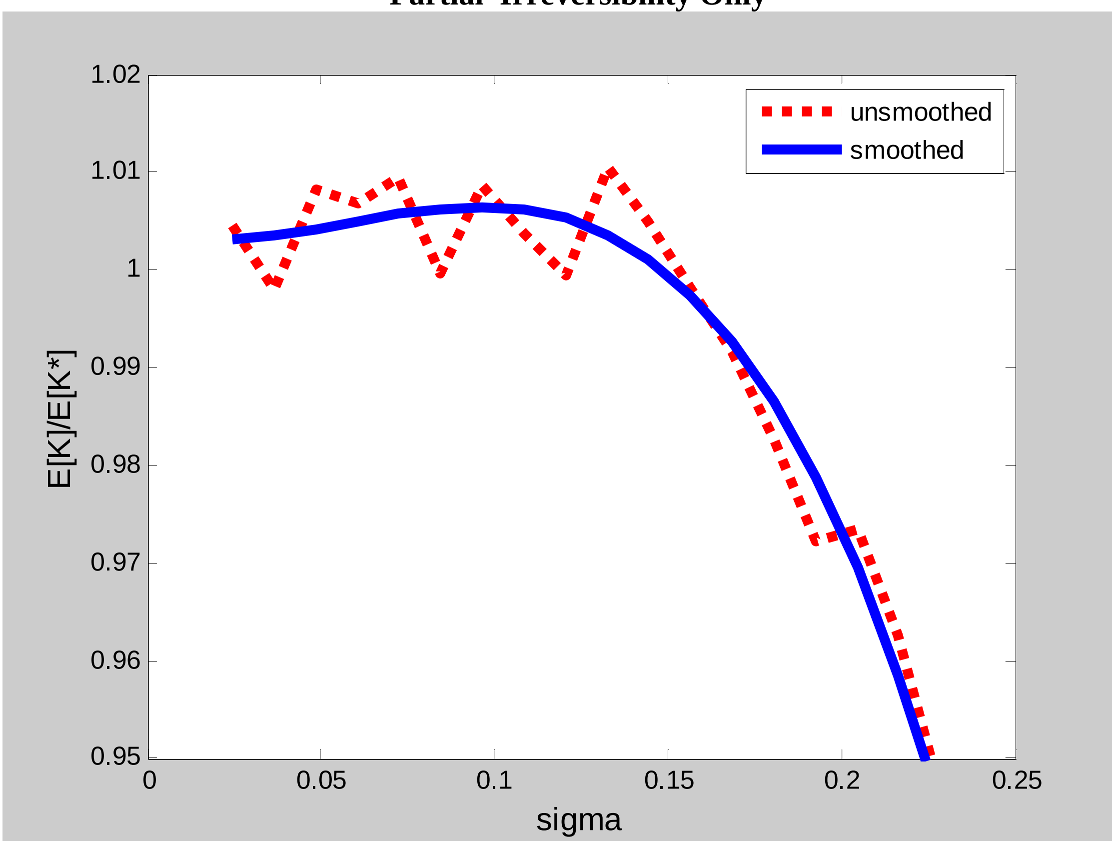

Figure 3: The Effect of Uncertainty on Average Capital Stock Levels: Fixed Adjustment Costs Only

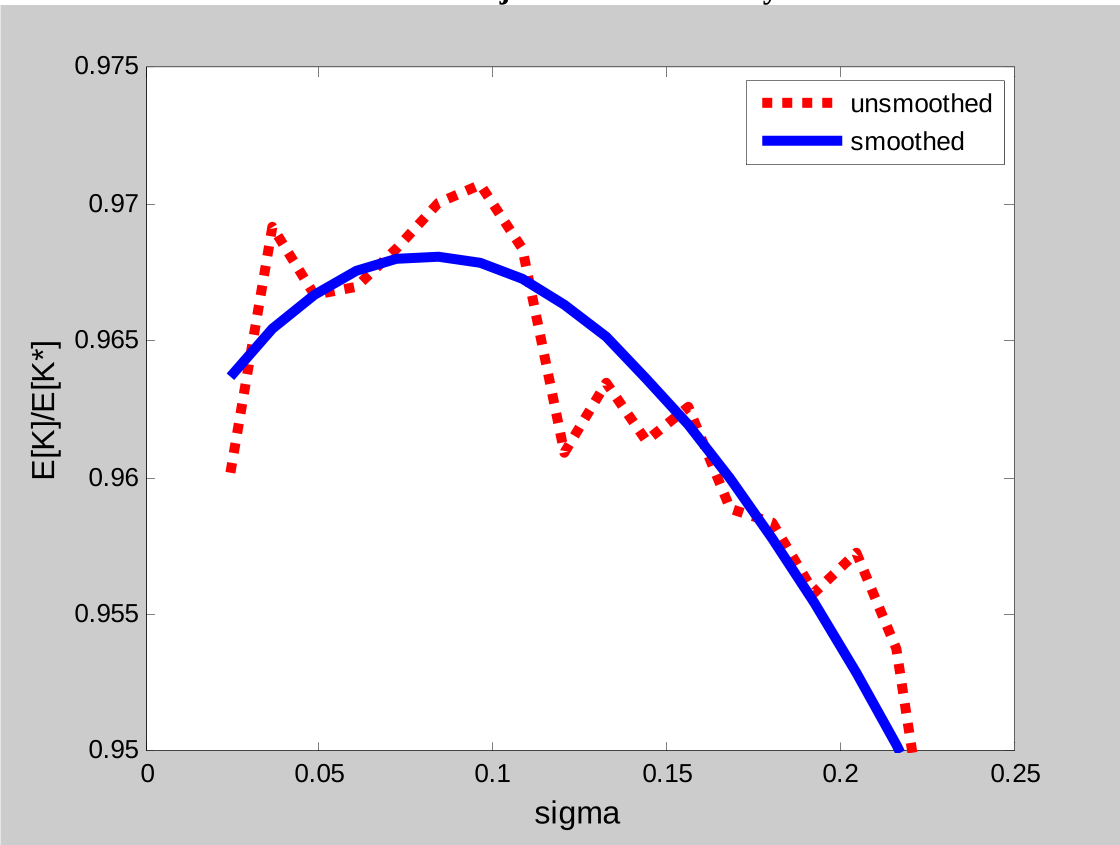

Figure 4: The Effect of Uncertainty on Average Capital Stock Levels: Quadratic Adjustment Costs Only